The Global COE Program
“The Next Generation of Physics, Spun from Universality and Emergence”
Bilateral International Exchange Program (BIEP, invite) report
Send report to: Your responsible Professor in Kyoto University
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(Year/Month/Day) 2009/02/05

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Research Project

Title	Thermal inflation and generation of gravitational waves
Duration	09. 1. 2 ~ 09. 2. 6 (35 days)

Please summarize your activities and results during your stay in Kyoto University. You can add a sheet, if you need more space. You can also write any comments and requests to the GCOE program. We will appreciate them.

Whatever it is primordial one or thermal one, there is a process called preheating after inflation. According to the thermal inflation model[1], the spectrum of perturbation seems to evolve into some asymptotic state. We can study this preheating problem analytically. On the one hand, we have a Lagrangian; so, it is easy to derive a field equation. And, we can deal with the equation perturbatively. However, the calculation became messy and as a result we lose the physical meanings.

On the other hand, there is a systematic way to account for all of these couplings, neglecting the information of the phase of each oscillation mode. Namely, we can write down a Boltzmann equation for smoothly distributed particles. Now, let’s consider the real classical flaton field which played a crucial role in the thermal inflation model. Then, we can write down the interaction terms.

$$H_{int} = \frac{1}{3}\lambda_3\phi^3 + \frac{1}{4}\lambda_4\phi^4 + \frac{1}{5}\lambda_5\phi^5 + \cdots$$

One may think that the Bose-Einstein distribution

$$f = \frac{1}{\exp[E - \mu/T] - 1}$$

will be realized. Or, in the classical field limit, the energy per each mode becomes the same and is

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[1] G. N. Felder, H. Kim, W. Park and E. D. Stewart, Preheating and Affleck-Dine leptogenesis after thermal inflation, 2007.

proportional to the temperature, which means an equipartition

$$f = \frac{T}{E - \mu}$$

is realized. Such a classical treatment breaks down at a quantum cutoff.

$$E - \mu \sim T$$

By applying the original Boltzmann equation to this classical field case, we can derive the time derivative of occupation number. I attached the part of the research note.

For the scalar CP invariant interaction

$$a + b + \ldots \longleftrightarrow i + j + \ldots \tag{17}$$

the Boltzmann equation is

$$\begin{aligned} \dot{f}_a &= \frac{1}{2E_a} \int d\Pi_b \, d\Pi_c \ldots d\Pi_i \, d\Pi_j \ldots (2\pi)^4 \, \delta(p_a + p_b + \ldots - p_i - p_j - \ldots) \\ &\quad \times |\mathcal{M}|^2 \left[f_i f_j \ldots (1 + f_a)(1 + f_b) \ldots - f_a f_b \ldots (1 + f_i)(1 + f_j) \ldots \right] \end{aligned} \tag{18}$$

where

$$d\Pi = \frac{1}{(2\pi)^3} \frac{d^3 p}{2E} \tag{19}$$

2.1 Classical field limit

For $f \gg 1$, Eq. (18) reduces to

$$\begin{aligned} \dot{f}_a &= \frac{1}{2E_a} \int d\Pi_b \, d\Pi_c \ldots d\Pi_i \, d\Pi_j \ldots (2\pi)^4 \, \delta(p_a + p_b + \ldots - p_i - p_j - \ldots) \\ &\quad \times |\mathcal{M}|^2 f_a f_b \ldots f_i f_j \ldots \left(\frac{1}{f_a} + \frac{1}{f_b} + \ldots - \frac{1}{f_i} - \frac{1}{f_j} - \ldots \right) \end{aligned} \tag{20}$$

2.2 Scattering

For

$$V = \frac{1}{2} m^2 \phi^2 + \frac{1}{3!} \lambda_3 \phi^3 + \frac{1}{4!} \lambda_4 \phi^4 + \frac{1}{5!} \lambda_5 \phi^5 + \ldots \tag{21}$$

the matrix element for $a + b \leftrightarrow i + j$ scattering is

$$|\mathcal{M}| = \begin{cases} \lambda_4 & \text{for} \quad a + b \leftrightarrow i + j \\ \dfrac{\lambda_3^2}{(p_a + p_b)^2 - m^2} & \text{for} \quad a + b \leftrightarrow \ldots \leftrightarrow i + j \end{cases} \tag{22}$$

What about t-channel $a + b \leftrightarrow a + i + \ldots \leftrightarrow i + j$? and the matrix element for $a + b \leftrightarrow i + j + k$ scattering is

$$|\mathcal{M}| = \begin{cases} \lambda_5 & \text{for} \quad a + b \leftrightarrow i + j + k \\ \dfrac{\lambda_3 \lambda_4}{(p_a + p_b)^2 - m^2} & \text{for} \quad a + b \leftrightarrow \ldots \leftrightarrow i + j + k \\ \dfrac{\lambda_3^3}{\left[(p_a + p_b)^2 - m^2 \right] \left[(p_a + p_b - p_i)^2 - m^2 \right]} & \text{for} \quad a + b \leftrightarrow \ldots \leftrightarrow i + \ldots \leftrightarrow i + j + k \end{cases} \tag{23}$$

In conclusion, we derived some first-order differential equations from Boltzmann equation. We will solve these equations numerically. Such thermodynamics of preheating is quite interesting since it can describe the evolution of statistical state of the universe.