Cross-cultural comparison between black hole perturbation and post-Newtonian theory

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Talk plan

Introduction

- Motivation
- Black hole perturbation theory
- Self-force picture in BH perturbation
- Strategy to calculate SF

Recent results

- SF corrections in periastron advance and red shift function
- Comparison to PN results
- Comparison to numerical result

Summary and future work

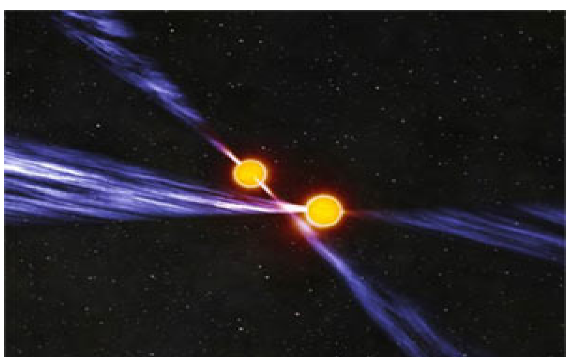
Motivation

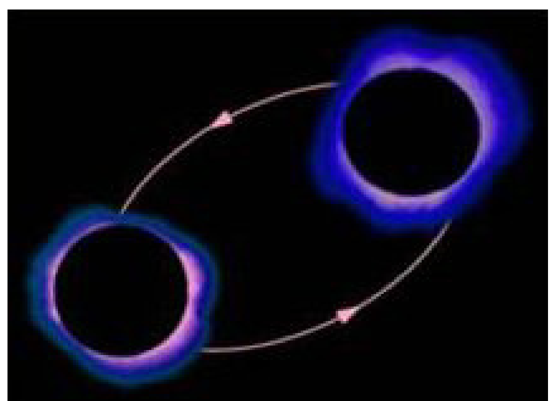
Two-body problem

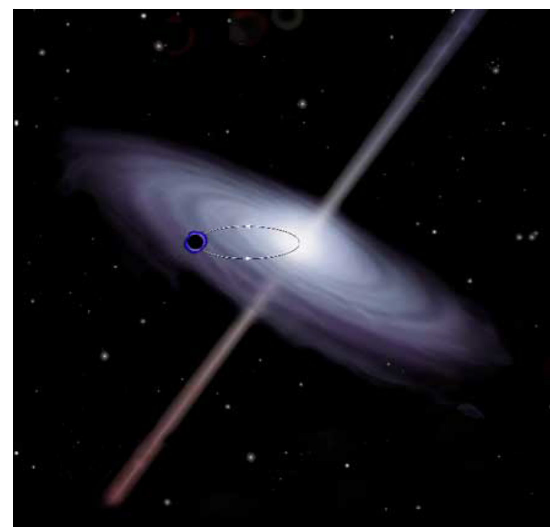
Newton	Solved exactly by reducing to one-body problem
GR	Difficult to solve exactly due to the non-linearity of GR and the “*radiation reaction*” problem

As targets of the gravitational wave observation

Well-motivated by attempts of the gravitational wave detection.
Understanding of orbital evolution and accurate prediction of waveform are necessary for the detection.

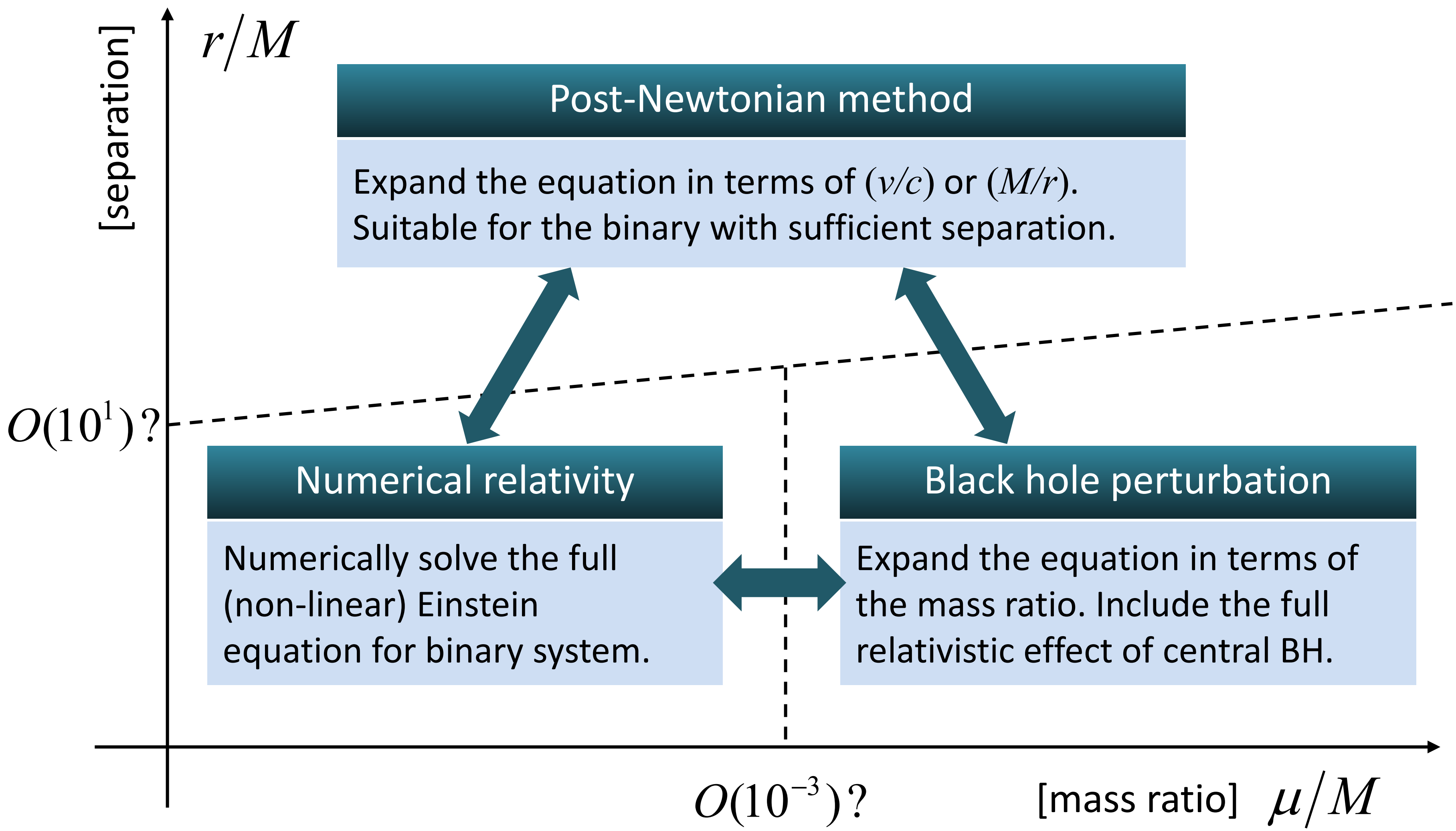

Compact binaries


Massive BH binaries


Extreme mass ratio inspirals

(From LISA website, http://lisa.nasa.gov/)

Approaches to two-body problem in GR



Black hole perturbation

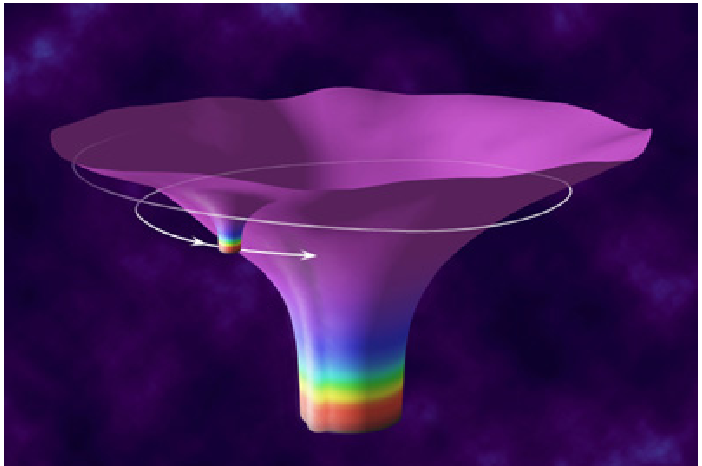
We deal with 2-body system by using black hole perturbation regime.

Spacetime

$$g_{\mu\nu} = g^{(b)}_{\mu\nu} + h_{\mu\nu}$$

Background of central BH

Perturbation induced by a particle



(From LISA website, http://lisa.nasa.gov/)

Linearized Einstein equation

$$\delta G^{(1)}_{\mu\nu}[h^{(1)}_{\mu\nu}] = 8\pi G T^{(1)}_{\mu\nu}$$ → Energy-momentum tensor for geodesic

$$\delta G^{(1)}_{\mu\nu}[h^{(2)}_{\mu\nu}] = 8\pi G T^{(2)}_{\mu\nu} + \delta G^{(2)}_{\mu\nu}[h^{(1)}_{\mu\nu}, h^{(1)}_{\mu\nu}]$$

$$\vdots$$

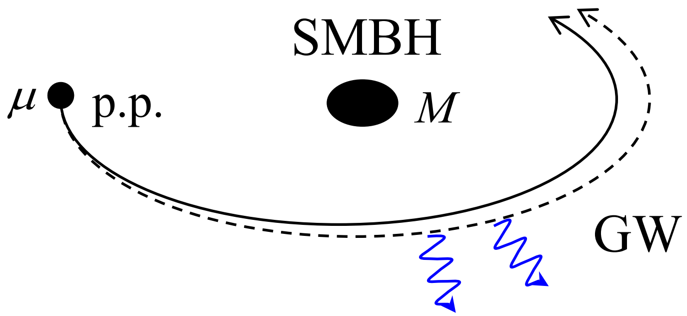
$\delta G^{(2)}_{\mu\nu}[h^{(1)}_{\mu\nu}, h^{(1)}_{\mu\nu}]$ → Non-linear effect

$8\pi G T^{(2)}_{\mu\nu}$ → Self-force effect

Self-force picture in BH perturbation

Motion of the particle

In test particle cases, the motion is described by the geodesic equation of the background.



$$\mu\left(\frac{d^2 z^\mu}{d\tau^2} + \Gamma^{(0)\mu}_{\alpha\beta}\frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau}\right) = 0$$

Including the back reaction due to the self-field

$$\mu\left(\frac{d^2 z^\mu}{d\tau^2} + \Gamma^{(0)\mu}_{\alpha\beta}\frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau}\right) = F^\mu$$

Self force $\propto \nabla h$

Correspond to the shift from the background geodesic.

To obtain the equation of motion, we need to calculate the metric perturbation and to derive the self-force from the MP.

Strategy to calculate the SF

Solve the geodesic equation and construct energy-momentum tensor.

$$\mu\left(\frac{d^2 z^\mu}{d\tau^2} + \Gamma^{(0)\mu}_{\alpha\beta}\frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau}\right) = 0 \quad \longrightarrow \quad T^{(1)}_{\mu\nu}(x,z)$$

Solve the field equation with the energy-momentum tensor.

$$\delta G^{(1)}_{\mu\nu}[h^{(1)}_{\mu\nu}] = 8\pi G T^{(1)}_{\mu\nu}$$

Construct the self-force from MP via ‘mode-sum’ scheme.

$$F^\alpha(\tau) = \sum_\ell \lim_{x\to z(\tau)} \left(F^\alpha_\ell[h_{\mu\nu}(x)] - F^\alpha_\ell[h^{\rm dir}_{\mu\nu}(x)]\right)$$

So far, we have developed a numerical code to calculate the self-force for general bound (eccentric) orbits in Schwarzschild spacetime.
(q.v. my previous reports)

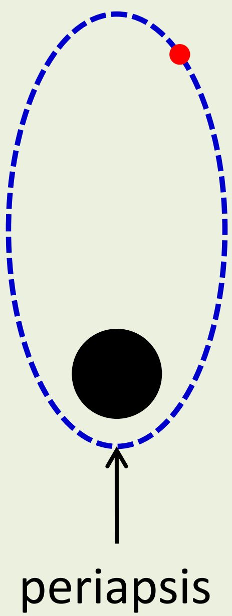
In this report, we investigate the SF effects on the particle’s motion and consider comparisons to other methods.

SF correction in periastron advance

Consider a particle (red point) orbiting a black hole (black circle) along an eccentric orbit (blue dashed line).

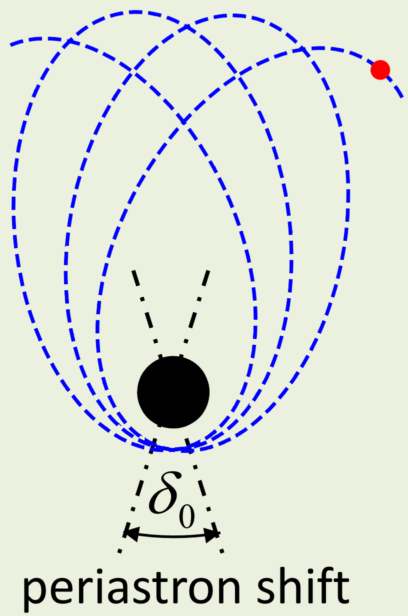
Newtonian

$$\frac{\Omega_\varphi}{\Omega_r} = 1$$



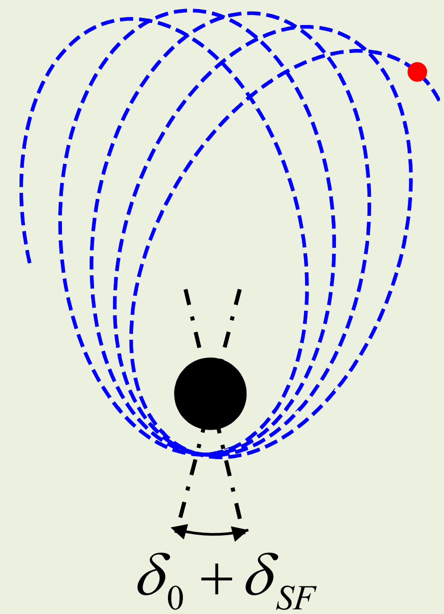
GR (geodesic)

$$\frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\delta_0}{2\pi}$$



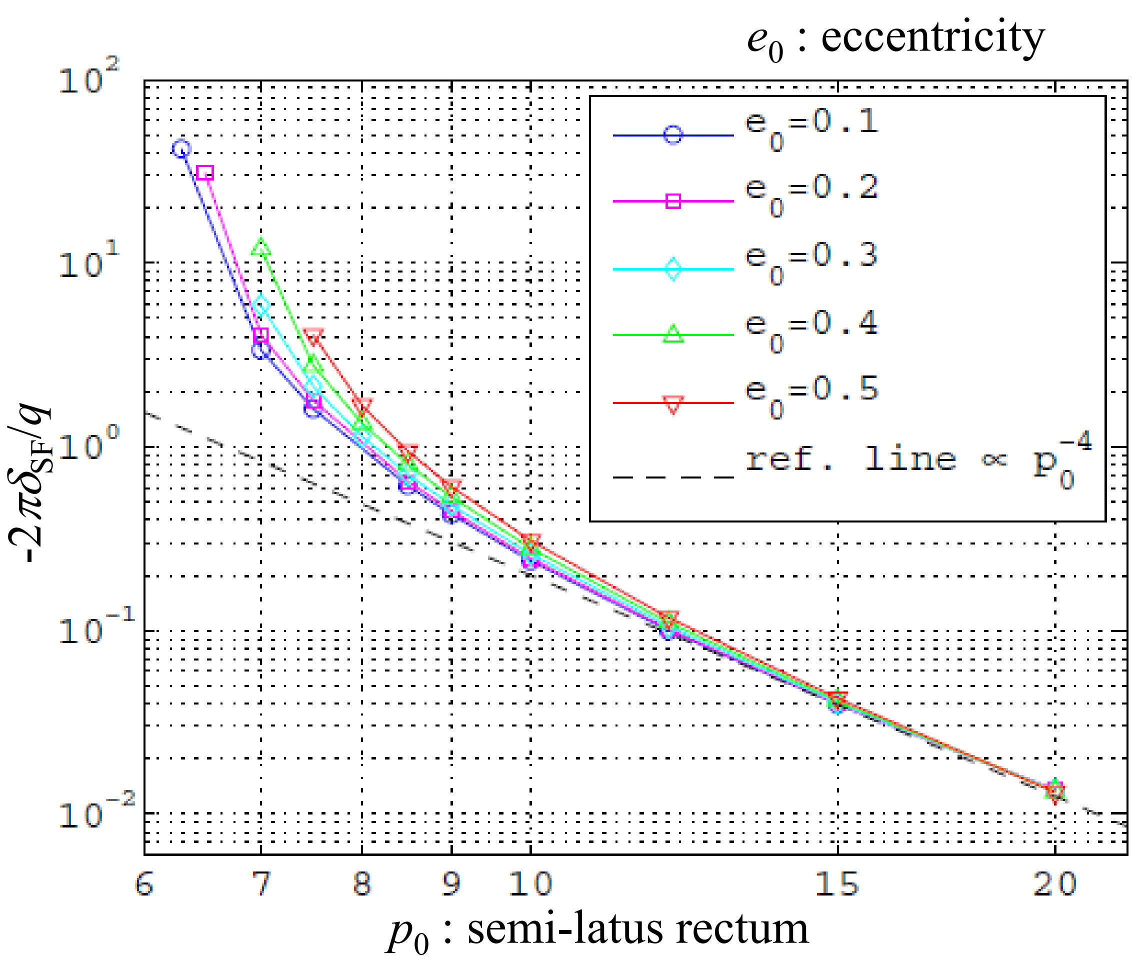
GR (SF included)

$$\frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\delta_0 + \delta_{SF}}{2\pi}$$



SF correction in periastron advance

[Barack and NS, 2011]



1. Negative for any (p_0 , e_0).

⟹ The SF acts to reduce the periastron advance.

2. Weak dependence on e_0 at large radius.

3. $\delta_{SF} \propto p_0^{-4}$ for large radius.

⟹ It falls off faster than the background advance, $\delta_0 \propto 1/p_0$.

4. $\delta_{SF} \propto 1/(p_0 - 6 - 2e_0)$ near the separatrix.

Comparison to PN results (I)

[Damour 2009]

Consider the ratio of two frequency at the circular limit:

$$W \equiv \lim_{e\to 0}\left(\frac{\Omega_r}{\Omega_\varphi}\right)^2 = 1 - 6x + \underline{\underline{q\rho(x)}} + O(q^2)$$

Ω_φ : orbital (azimuthal) frequency
Ω_r : radial frequency
$x \equiv [(\mathrm{M}+\mu)\Omega_\varphi]^{2/3}$

- related to the SF correction in periapsis advance
- gauge-invariant given as a function of gauge-invariant parameter, "x"

Post-Newtonian formula of ρ for circular limit:

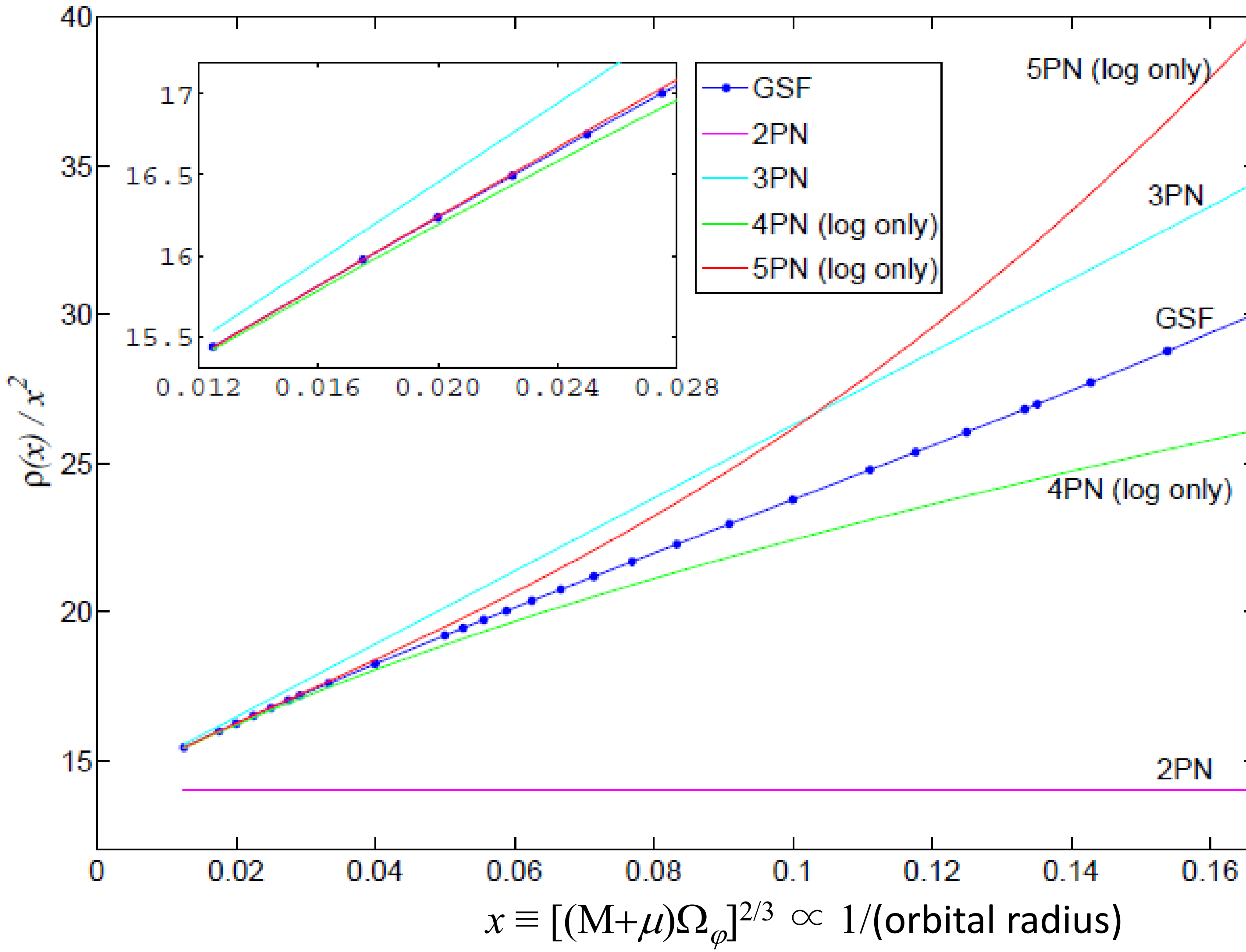
$$\rho(x) = \rho_2 x^2 + \rho_3 x^3 + \left(\rho_4^c + \rho_4^{\log}\ln x\right)x^4 + \left(\rho_5^c + \rho_5^{\log}\ln x\right)x^5 + \left(\rho_6^c + \rho_6^{\log}\ln x\right)x^6 + O(x^7)$$

Known parameters

$$\rho_2 = 14, \quad \rho_3 = \frac{397}{2} - \frac{123}{16}\pi^2, \quad \rho_4^{\log} = \frac{16}{15}\times 157 = 167.466666\cdots, \quad \rho_5^{\log} = -\frac{11336}{7} = -1619.428571\cdots$$

Comparison to PN results (I)

[Barack, Damour and NS, 2010]



Calibration of unknown PN parameters of $\rho(x)$

[Barack, Damour and NS, 2010]

Source of errors in fitting SF data to PN formula

- Numerical error in computing the SF
  → Improving the computational accuracy is required.
- Incompleteness of fit model with finite PN order
  → To reduce this error, we try to fit SF data to several PN models and take the "best guess" values of the parameters.

"Best guess" of unknown PN parameters

$$\rho_4^c = 69^{+7}_{-4}, \quad \rho_5^c = -4800^{+400}_{-1200}, \quad \rho_6^{\log} > 0$$

We can use these results to put some constraints to some analytic models.
(e.g. Effective one body)

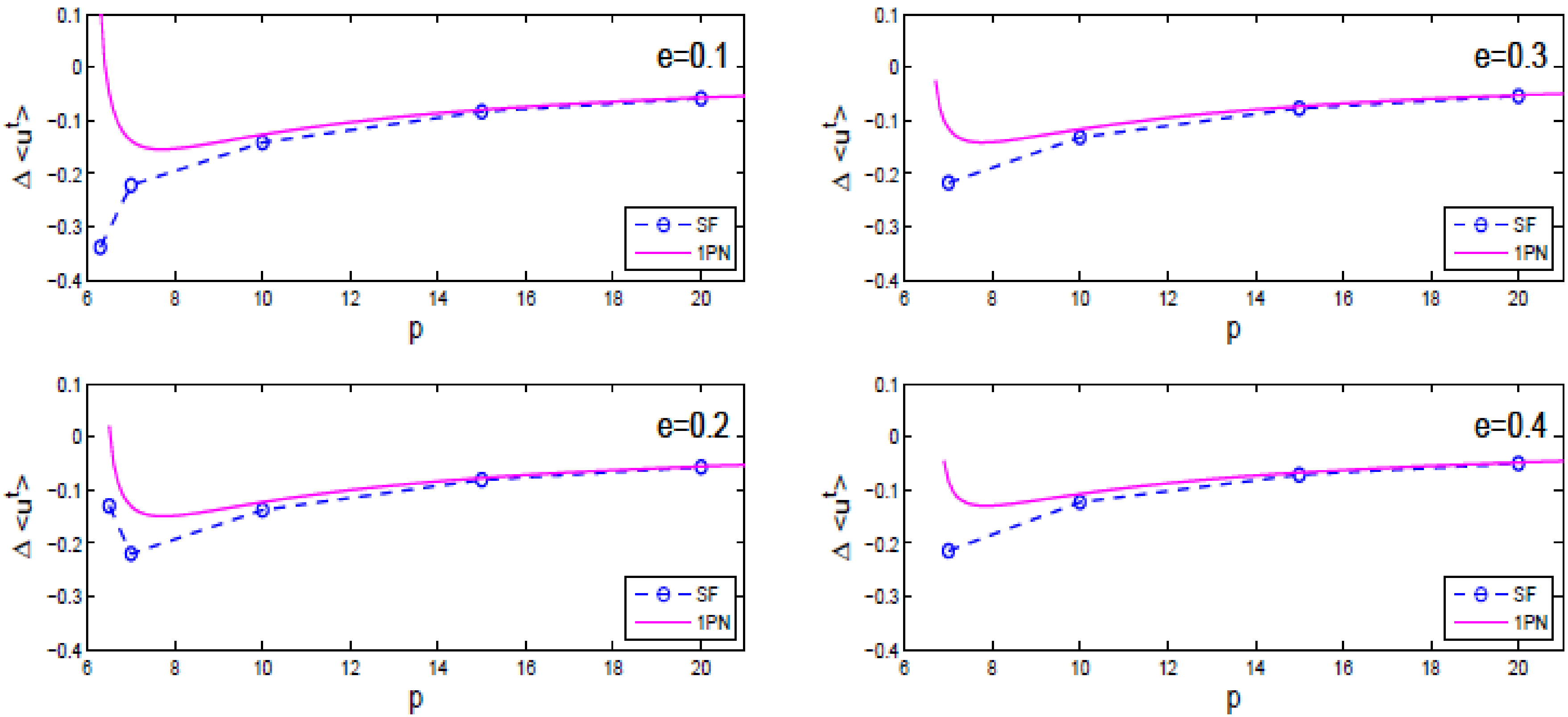
Our current SF data gives only very rough estimate of unknown parameters.
The higher PN parameters cannot be constrained due to the noise.
To reduce the noise, we need more accurate computation of the SF.

Comparison to PN results (II)

[Barack, LeTiec and NS, in preparation]

Consider the SF correction in t-component of the 4-velocity of the particle.

$$\left\langle u^t \right\rangle_t \equiv \frac{1}{T_r}\int_0^{T_r} u^t\, dt$$: averaged over one radial period.

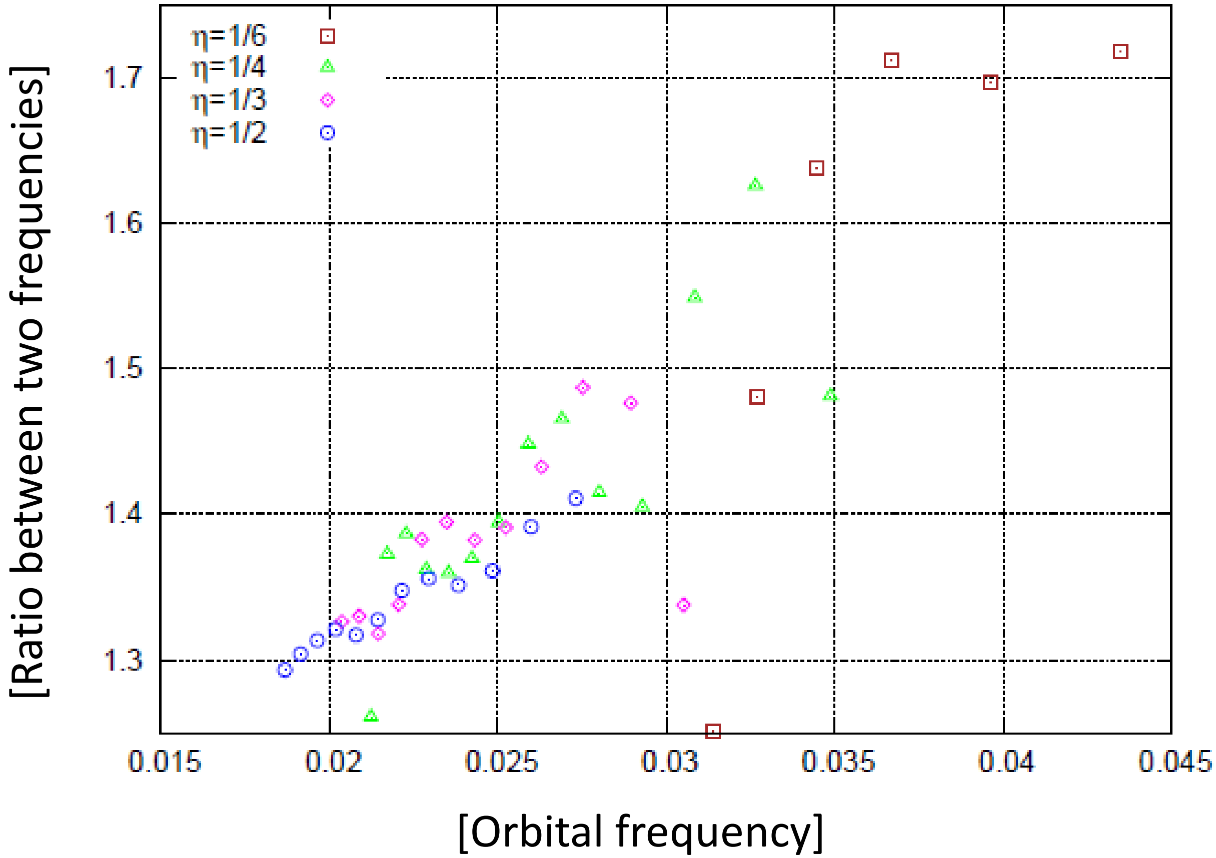


As expected, we can find good agreement with 1PN results at large radius.

Comparison to NR results

[Mroue et al, 2010]

Consider the relation between Ω_φ and $\Omega_\varphi / \Omega_r$ at circular limit.
Mroue et al. obtain the relation in fully non-linear simulations of BH binary.



Blue : $q=1/2$
Pink : $q=1/3$
Green : $q=1/4$
Brown : $q=1/6$

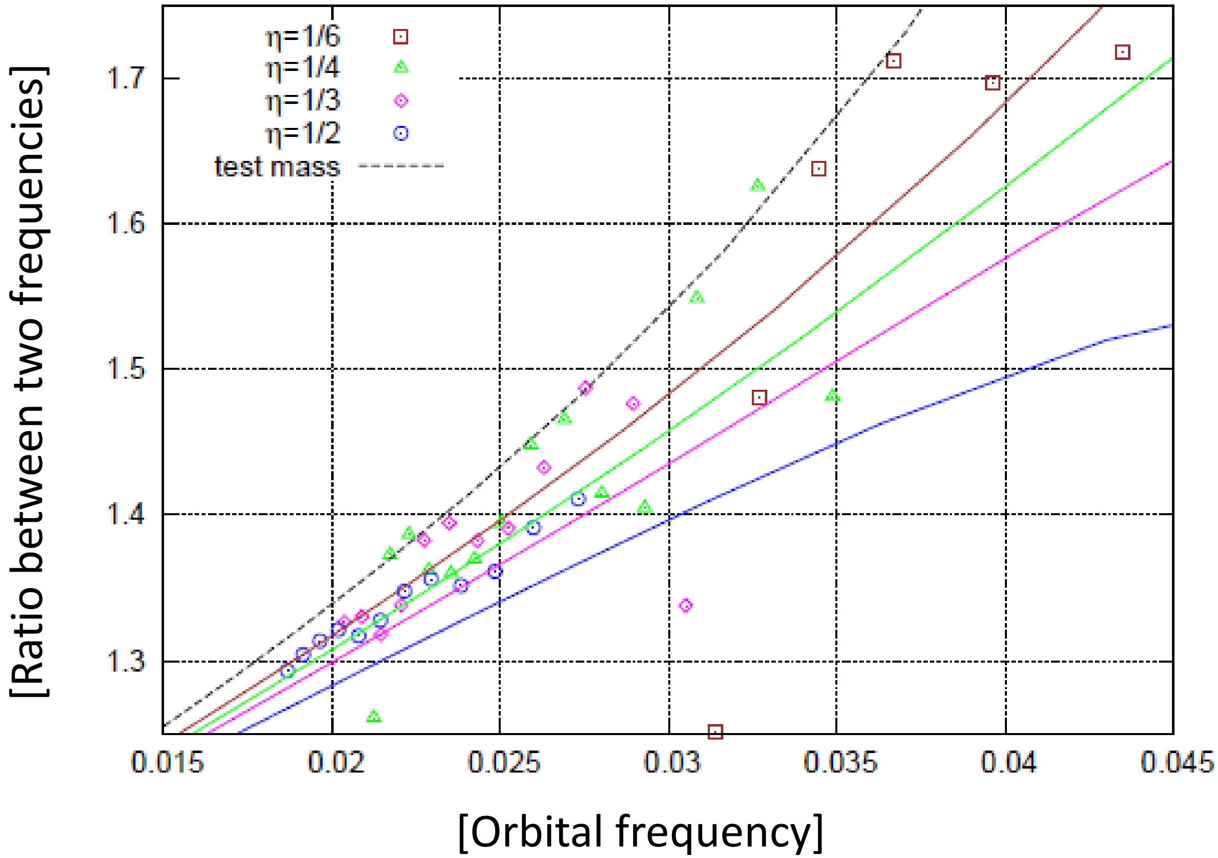
q = (mass ratio)

Comparison to NR results

[Barack and NS, 2011]

Consider to compare our SF results with the NR results.

⟹ A detailed comparison is difficult due to the large error of NR results.



Dashed : Geodesic case
Solid line : SF prediction
Data points : NR results

Blue : $q=1/2$
Pink : $q=1/3$
Green : $q=1/4$
Brown : $q=1/6$

$q=$ (mass ratio)

Summary

This work

- Estimate the self-force effects on eccentric orbits in Schwarzschild spacetime. (periastron advance and t-component of 4-velocity)
- Compare our self-force results to the post-Newtonian ones.
- Calibrate the unknown PN parameters from the SF data
- Compare our results to numerical relativity (Preliminary)

Future work

- More accurate data to make an analytic model
- Orbital evolution with the self-force effect.
- SF effect on the waveform.
- Second order perturbation (with SF and non-linear effects)