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京 都 大 学 大 学 院 理 学 研 究 科

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要旨集

２０２３年１１月２４日（金）

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Nonlinear optical response in strongly correlated electron systems

Condensed Matter Theory Group Akira Kofuji

Abstract We have studied the gap dependence of high-order harmonic generation in two-level systems and found an unconventional gap dependence in the strongly light-matter coupled regime. Also, we have investigated two-particle correlation effects on nonlinear optical phenomena and revealed that two-particle fluctuations beyond the mean-field level significantly enhance the photocurrent.

Nonlinear responses have been widely studied because of their great potential to reveal states in materials, such as ferroelectric domains[1], Berry curvature dipole in topological materials[2], and so on. Recently, nonlinear responses in strongly correlated electron systems have also attracted much attention, and various experiments have been conducted[3,4,5]. For example, in a Mott insulator Ca_2RuO_4, an unconventional gap dependence of High harmonic generation(HHG) has been observed[3], which cannot be understood by the conventional three-step model. Also, in 1d ferroelectric organics $(TMTTF)_2X$(X=AsF_6 and PF_6), a photoinduced increase of short-range correlations has been observed[4], and, in a chiral magnet MnSi, nonlinear nonreciprocal transport is enhanced above the transition temperature[5]. All these experiments imply a close relationship between interactions and nonlinear responses. However, most theoretical studies on nonlinear responses rely on independent particle approximation[6], and to what extent interactions and fluctuations affect nonlinear responses are not fully understood.

In this presentation, first, we discuss the gap dependence of High harmonic generation in two-level systems without interactions[7]. The simplicity of a two-level system makes it possible to investigate its gap dependence in broad parameter regions varying the light-matter coupling and the rate of relaxation. As a result, we find that when the Rabi frequency is sufficiently large compared to the gap width, the HHG intensity grows as the gap width is increased, and the enhancement rate obeys a scaling law, which is independent of the incidental frequency. Also, we find that relaxation processes are not essential but increase the visibility of the gap dependence.

Next, we discuss the dynamics of two-particle correlations and their connection with nonlinear responses in the 1D interacting Rice-Mele model[8]. We utilize the correlation expansion method by Fricke[9] and calculate the nonlinear conductivity and the time evolution of two-particle correlations in a unified framework. As a result, we find that two-particle fluctuations can markedly enhance the nonlinear photovoltaic effect and, on the other hand, suppress the linear optical conductivity slightly. Furthermore, in the photoinduced steady state, the short-range two-particle fluctuation increases compared to that in the equilibrium. We will discuss how the interaction is related to these results by transforming the interaction to the band basis and considering corresponding two-particle correlations.

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Terahertz Doppler-free spectroscopy in rotational levels of gas-phase molecules

Solid State Spectroscopy Group Kohei Eguchi

Abstract We applied Doppler-free spectroscopy to rotational transitions of acetonitrile in the terahertz region. Pressure dependence of the homogeneous width was confirmed beyond the Doppler-limit. The homogeneous width in the zero-pressure limit was 32 kHz and should be originated from transit-time broadening (~36 kHz) and lifetime broadening.

Rotational transition of gas-phase molecules appears in the terahertz (THz) frequency region. It is specific to the molecules, so called fingerprint spectrum and widely used for material analysis and for frequency calibration in fields ranging from chemistry and biotechnology to astrophysics: Numerous lines are listed in databases such as NASA JPL [1] and HITRAN [2]. On the other hand, due to the lack of high-power, narrow-linewidth terahertz light sources, high-precision spectroscopy methods developed in the electronic transition region have rarely been applied to rotational transitions [3, 4], and their early realization has been awaited.

In this study, we realized THz Doppler-free spectroscopy, a nonlinear spectroscopy method that observes a homogeneous width narrower than the Doppler-width, using an ultranarrow-linewidth THz source less than 1 Hz, and applied it to rotational transitions of gas-phase acetonitrile molecules. Since the available THz power is less than 10 μW, acetonitrile molecules with large oscillator strength of rotational transitions were used to realize the saturation region.

Figure 1 shows Doppler-free spectra for several sample pressures measured at room temperature. A narrow dip structure was observed at the center of the Doppler-broadened (~600 kHz) peak. The pump laser power was 6 μW which was 120 times less than the power in the previous study [3]. We determined pressure dependence of homogeneous width (Γ) as shown in Fig. 2. The linear dependence suggests the collisional broadening. We found that the homogeneous width in the zero-pressure limit is 32 kHz and should be originated from the transit-time-broadening (~36 kHz) and lifetime broadening [5].

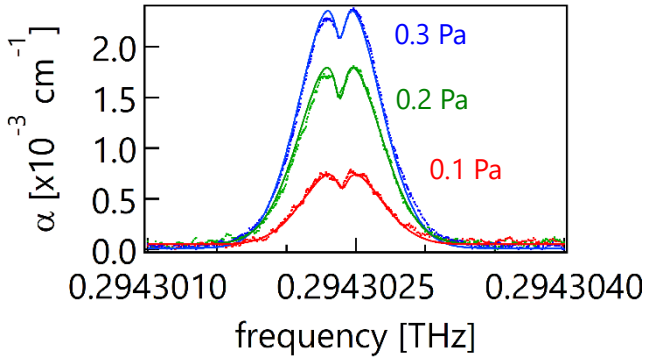

Fig. 1: Pressure dependence of the absorption spectrum of (J, K) = (15, 0) → (16, 0) rotational transition in acetonitrile under THz pumping in several pressures. dots: measured data, lines: fitting curves.

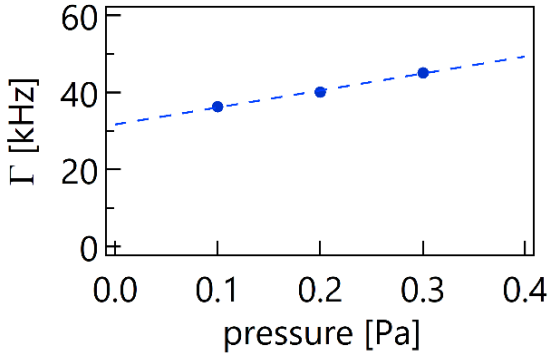

Fig. 2: Pressure dependence of homogeneous width. Closed circles: homogeneous width obtained by the fitting (shown in Fig. 1). dashed line: linear fitting curve.

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Wavenumber of Self-organized Stripes in Liquid Crystal Gels

Soft Matter Physics Laboratory　　Akinori OOKA

Abstract In liquid crystal gels, self-organized stripe domains appear in response to external stimuli. To determine what governs stripe width, anisotropy in polymer steps and surface anchoring strength were varied, leading to significantly altered stripe widths. A comparison with theory suggests that liquid crystal elasticity is more dominant than previously believed.

Liquid crystal nematic gel (LCG) has a structure in which a polymer network is swollen with liquid crystal molecules. The polymer chain and the liquid crystal interact, leading to polymer chain rearrangement when the director changes [1]. It is known that both the elasticity of the polymer network and the Frank elasticity of the liquid crystal govern the physical properties of the gel. The former becomes dominant when the liquid crystal director is heterogeneous; the opposite is true for the latter. Due to this competition, self-organized stripe domains appear in LCG under certain external stimuli due to non-uniform rotation of the director [1]. The free energy and wavenumbers of such a striped LCG have been calculated based on this picture [2], but their quantitative accuracy compared to experiment remains unexplored.

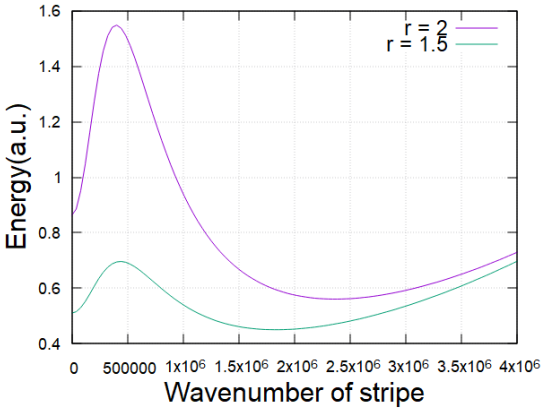

Fig.1：Change in free energy due to polymer anisotropy *r*

The purpose of this study is to experimentally determine the contribution of polymer chain and liquid crystal elasticity to stripe formation. Specifically, I prepared and observed LCGs and investigated how variations in polymer chain anisotropy and surface anchoring strength lead to changes in stripe wavenumber.

Firstly, LCGs were produced with different polymer chain anisotropy. LCGs polymerized at lower temperatures lead to polymer networks with more anisotropy in polymer segment step direction [1]. Anisotropy r of the chain is 1 when the polymer chain is isotropic and increases with increasing anisotropy in the direction of the initial director. In theory, increasing the anisotropy *r* of the chain alone should increase the wavenumber of the stripes (Fig.1). The opposite was true in experiments. One possible cause of this is that the polymer chains become stretched and hardened, making it difficult for the liquid crystal molecules to rotate.

This leads me to believe that the ease of rotation of the director has a more significant effect on stripe formation than expressed in theory. To confirm this, I repeated the experiments in cells with different in-plane anchoring strength in the cell surface: this led to a reduction in stripe wavenumber. The ease of rotation, as captured in a modified effective cell thickness, could be incorporated into the theory, but again, we found that the experimental result contradicted predictions. It is clear that the ease of rotation of the director has a greater effect than the anisotropy of the polymer chain.

Both experimental results compared to existing theory leads me to believe that the contribution of the elasticity of the liquid crystal is larger than that expected in theory.

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Theoretical Study of Invertible States Using Matrix Product States

YITP Shuhei Ohyama

Abstract
The Berry phase, introduced by Michael V. Berry in 1984, has been widely applied in the definition of topological invariants, inspired by the trend of topological phases originating from quantum Hall systems. We discussed the formulation of higher Berry phases in 1+1-dimensional systems. Specifically, by employing the matrix product state, we define the inner product for three states and demonstrate how this quantity provides a natural definition of the higher Berry phase.

An invertible state refers to a generic term for states with short-range correlations that are realized as the ground state of a unique gapped Hamiltonian. Under appropriate symmetries, invertible states exhibit non-trivial quantum phases, which are referred to as Symmetry-Protected Topological (SPT) phases. It is also known that considering a family of invertible states gives rise to topological transport phenomena similar to the Thouless pump. The invertible phase, that is, the phase constituted by invertible states, is known to have a rich structure and has been actively researched in the contexts of condensed matter physics, quantum information theory, particle physics, and mathematical physics.

Such topological phases arise because the space formed by all invertible states possesses a non-trivial topology. Therefore, by defining topological invariants for families of invertible states, it is possible to provide a unified classification for SPT phases and the classification of Thouless-like pump phenomena. However, aside from special cases such as free fermion systems, no method for constructing such invariants had been known.

A crucial quantity in constructing the invariants is the Berry phase. However, when naively calculating the Berry phase for high-dimensional many-body Hamiltonians, a divergence problem arises in the thermodynamic limit, necessitating an appropriate generalization of the definition of the Berry phase. To address this, we used a Matrix Product State (MPS) representation.

As a result, by employing formulations known as fermionic MPS and G-injective MPS, we discovered invariants that characterize the fermion parity pump and G-charge pump. In particular, using these, we demonstrated that the fermion parity pump, known to exist in free fermion systems, is stable under interactions [1].
Furthermore, by using injective MPS, we defined a generalization of the Berry phase in 1+1 dimensional systems as an overlap of three quantum states. We showed that by integrating this, invariants for the family can be extracted [2,3,4].

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Luminescence fine structure and exciton–phonon interactions in single halide perovskite nanocrystals

Optical Materials Science Group Kenichi Cho

Abstract We studied photoluminescence spectra of single perovskite nanocrystals at low temperatures. We observed the luminescence fine structure and determined its origin and the strength of exciton–phonon coupling. We found that the exciton–phonon coupling is strongly affected by the spatial overlap of electron and hole wavefunctions.

Lead halide perovskites ABX_3 (A = Cs, MA (CH_3NH_3), FA ($HC(NH_2)_2$), B = Pb, and X = Cl, Br, I) are a new class of direct bandgap semiconductors. They possess excellent optical properties such as large absorption coefficients, high photoluminescence (PL) quantum yields, and long carrier lifetimes, which are suitable for optoelectronics applications [1]. Halide perovskite nanocrystals (NCs) have a well-defined cubic shape with a narrow size distribution [2]. Quantum confinement effects in NCs increase the PL quantum efficiency with keeping the superior materials properties of their bulk counterparts. Thus, halide perovskite NCs have attracted worldwide attention as materials for light-emitting diodes and lasers.

In this study, we examine the PL spectra of single $CsPbBr_3$ and $FAPbBr_3$ NCs by single-dot spectroscopy and discuss the origin of the luminescence fine structure in the perovskite NCs. The measurements were performed at low temperatures, where the narrow PL linewidth of single NCs is suited to study the luminescence fine structure of NCs. The PL linewidth of all inorganic $CsPbBr_3$ NCs was observed to be narrower than that of organic-inorganic hybrid $FAPbBr_3$ NCs. In addition, the PL spectra in $CsPbBr_3$ NCs have different exciton peak structures and can be characterized by structures with one, two, or three exciton peaks, reflecting the bright triplet excitons in the perovskite NCs [3,4]. Several PL peaks appear in the low energy side of the strong exciton peaks. From the size-dependence of the peak energies and the excitation fluence dependence of the PL intensities, we clarified that the low-energy PL peaks originate from trions, biexcitons, and longitudinal optical (LO) phonon replicas of exciton PL. The binding energies of trions and biexcitons increase with decreasing NC size. The LO phonon energies are independent of NC size, but the Huang–Rhys factor of excitons, i.e., the strength of the exciton–phonon coupling, drastically increases with decreasing NC size. We concluded that the Huang–Rhys factor reflects the spatial overlap of the electron and hole wavefunctions inside the NCs [3,4]. The temperature dependence of the PL linewidth was also measured from 5.5 to 200 K. The temperature-dependent broadening of the PL linewidth is mainly determined by the Fröhlich interaction between excitons and LO phonons. The exciton–LO-phonon coupling constant and effective LO phonon energy show no changes even when the temperature is varied across the orthorhombic-tetragonal crystal phase transition temperature. These results demonstrate that the exciton–phonon coupling in the perovskite NCs are not affected by the differences in crystal structures [5].

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Impact of intraband transitions in high-order harmonic generation from solids

Optical Materials Science Group Kotaro Nakagawa

Abstract We have studied nonlinear coupling between intraband and interband transitions in high-order harmonic generation (HHG). By observing HHG from semiconductor nanocrystals with different diameters, we concluded that intraband transitions with discrete electronic states determine the size-dependent HHG intensity.

The recent development of laser technology has made it possible to generate optical pulses with high electric field strength reaching the magnitude of the Coulomb field between an electron and an atomic nucleus in solids. Such intense laser pulses have provided opportunities to investigate completely new phenomena, and have been attracting intensive studies on strong-laser-field physics. One of the most important nonlinear phenomena is high-order harmonic generation (HHG), where the fundamental light with a frequency ω generates its harmonics, i.e., higher energy lights with frequencies of $n\omega$ (n is integer). HHG is attracting great interest as a novel light source in the extreme ultraviolet region, which enables attosecond pulse generation. To realize highly efficient HHG, intraband and interband transitions in solids have been intensively studied [1,2]. The former corresponds to the intraband acceleration of carriers driven by a laser electric field; the latter corresponds to the optical excitation of carriers between the conduction and valence bands. However, previous experiments have been unable to clarify how these two transitions affect HHG. To achieve high-efficiency HHG, it is indispensable to gain deeper insight into the role of these two transitions.

In this study, we focused on HHG from semiconductor nanocrystals, whose electronic structure can be changed from discrete to continuous by controlling the nanocrystal size. This tunability of the electronic structure makes nanocrystals well-suited for manipulating the intraband transitions. HHG from CdSe nanocrystal thin films was observed under mid-infrared pulse excitation. Figure 1(a) compares HHG spectra from CdSe nanocrystals with diameters of 2.8 and 3.8 nm. The odd harmonics between the 7th and 13th orders are clearly observed for the larger nanocrystals. We observed for the smaller nanocrystals that the HHG intensities are weaker at every harmonic order. In Fig. 1(b), we plot the dependence of the 7th harmonic intensity from CdSe nanocrystals against the nanocrystal diameter. It can be seen that as the nanocrystal diameter is decreased, the HHG intensity is rapidly suppressed at diameters around 2.5 nm. This drastic change can be explained by the increase in the gap between discrete energy levels originating from the quantum confinement effect (Fig. 1(b), inset) [3].

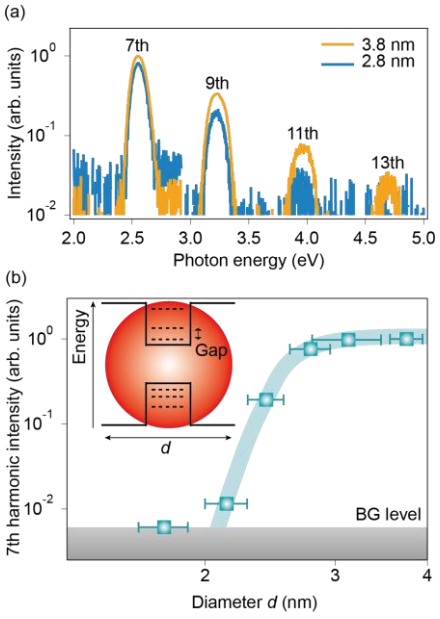

Fig. 1: (a) HHG spectra of CdSe nanocrystals. (b) Diameter dependence of the 7th harmonic intensity of CdSe nanocrystals. The inset shows energy-level diagram of nanocrystals.

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Bulk-boundary correspondence in non-Hermitian point-gap topological phases

Yukawa Institute for Theoretical Physics, Daichi Nakamura

Abstract We establish the bulk-boundary correspondence in topological phases that are intrinsic to non-Hermitian systems. As an application, we propose experimental platforms for these novel phases by coupling Hermitian topological insulators/superconductors to the environment.

There is a growing interest in the study of nonequilibrium physics. Among current trends, one of the noteworthy areas of research is the exploration of band topology in non-Hermitian systems. A remarkable characteristic of non-Hermitian systems is the existence of two distinct types of topological phases [1,2]. One type generalizes Hermitian topological phases, while the other is intrinsic to non-Hermitian systems, referred to as "line-gap topological phases" and "point-gap topological phases," respectively. While the bulk-boundary correspondence [3], in which bulk topological invariants count the number of gapless boundary states, is a fundamental principle in the former phases, topological phenomena in the latter have remained unclear. This issue stems from a novel boundary phenomenon with no Hermitian counterparts known as the non-Hermitian skin effect [4], where a significant number of bulk states become localized at the boundary. Although previous studies have shown that specific 1D point-gap topologies initiate this effect [5,6], understanding the precise phenomena linked to point-gap topological phases in each symmetry class and dimension has largely remained elusive. Furthermore, it has been uncertain whether the bulk-boundary correspondence exists in point-gap topological phases.

In this talk, we reveal the corresponding phenomena in certain symmetry classes, specifically $AZ^{\dagger}$ symmetry classes, and establish a theory of the bulk-boundary correspondence in point-gap topological phases [7]. As an application of our theory, we also discuss that Hermitian topological insulators/superconductors could implement point-gap topology by coupling to the environment [8].

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Topology and Strong correlation effect of Hidden symmetry breaking superconductor

Condensed Matter Theory Group Kosuke Nogaki

Abstract Hidden symmetry breaking has been attracted much attention due to the discovery of $CeRh_2As_2$. In this work, we clarified topological property and the role of strong correlation effect of this heavy fermion compound. We also investigated a novel mechanism of the field-induced superconductivity.

Symmetry stands as a pivotal concept in modern physics. In heavy fermion systems, there is an inherent hidden symmetry breaking tied to their crystalline structure. When combined with spin-orbit coupling, this hidden symmetry breaking influences the electronic band structure. The discovery of the multi-phase superconducting diagram in $CeRh_2As_2$ has carved a new path for exploration [1]. While foundational theories surrounding the multi-phase diagram have been suggested for over a decade [2], deeper research into topological properties and the effects of strong correlations has been keenly anticipated. In our study, we shed light on these facets of $CeRh_2As_2$, and we present a theoretical proposal on field-induced superconductivity unique to superconductors with hidden symmetry breaking. Detailed summaries of each aspect of our study are presented below.

***1. Topological crystalline superconductivity in locally noncentrosymmetric $CeRh_2As_2$* [3]**

Firstly, we elucidated the topological superconductivity in $CeRh_2As_2$. Assuming an odd-parity state in the high-field superconducting phase, we derived a Fermi-surface formula for Z_2 invariants [4]. These invariants specify the topological crystalline superconductivity, which is protected by the nonsymmorphic glide symmetry. Secondly, we performed a first-principles calculation on the electronic structure of $CeRh_2As_2$. By integrating these results, we evaluated the Z_2 invariants and identified the presence of topological superconductivity.

***2. Even-odd parity transition in strongly correlated locally noncentrosymmetric superconductors: Application to $CeRh_2As_2$* [5]**

While there are qualitative similarities between the weak-coupling theory [2] and the experimental results [1], inconsistencies exist. Specifically, the parity transition field derived from the theory is smaller by a factor of five. We have delved into the effects of strong correlations in $CeRh_2As_2$. Our findings indicate an *XY*-type magnetic fluctuation, aligning with recent NMR measurements [6]. Moreover, our theoretical results reveal that the parity transition field is considerably augmented due to electron correlation effects

***3. Field-induced superconductivity mediated by odd-parity multipole fluctuation* [7]**

We concentrate on the importance of inter-sublattice hopping, which supports odd-parity multipole fluctuation. Under the magnetic field, sublattice antisymmetric pairing is stabilized by odd-parity multipole fluctuation. This, in turn, amplifies the stability of the odd-parity superconducting state. Consequently, the field reentrant superconducting phase diagram can be attained. The aforementioned mechanism is an exclusive characteristic of strongly correlated electron systems. This discovery could furnish a new approach to comprehending exotic heavy fermion superconductors, such as UTe_2 and UCoGe.

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Quantum algorithm for persistent Betti numbers and topological data analysis

YITP, Quantum Information　Ryu Hayakawa

Abstract Topological data analysis (TDA) is an emergent field of data analysis based on computational topology. In TDA, computation of the persistent Betti numbers plays a central role. We present a first quantum algorithm for estimating the normalized persistent Betti numbers, which is exponentially faster than the known best classical algorithm.

Quantum computing is believed to outperform classical computing. It is crucial to understand in what applications quantum computing offers speedups and machine learning tasks are thought to be such promising applications of quantum computing. In classical computing, there is an emergent and rapidly growing field of data analysis called the *Topological Data Analysis* (TDA). TDA reveals the "shape of the data" through geometrical and topological analysis based on the mathematical framework called persistent homology. A crucial property of persistent homology, which is favorable in practical situations, is its stability against the noise concerning how the data is generated. However, classical algorithms do not work efficiently in the high-dimensional regime of TDA because there can be exponentially many elements that would contribute to the topology. To overcome this limitation, Lloyd et al.[1] first provided a quantum algorithm for estimating the normalized Betti numbers, which represent the number of (high-dimensional) holes of the data. However, there has been no known quantum algorithm for estimating the *persistent* Betti numbers, which are the central topological invariants used in TDA based on persistent homology.

In this work, we show a first quantum algorithm that can estimate the normalized persistent Betti numbers of arbitrary dimensions[2]. Our quantum algorithm is exponentially faster than the known best classical algorithm. Moreover, there is evidence for the exponential quantum advantage based on a computational complexity-theoretic assumption[3]. Our quantum algorithm is based on a positive-semidefinite hermitian operator called the persistent Laplacian. We estimate the normalized persistent Betti numbers by the estimation of the low-energy density of the persistent Laplacian. Figure 1 shows a quantum circuit for the quantum algorithm. We believe that our work opens a wide possibility of application of quantum computing for data analysis and machine learning based on the complex topology of the data.

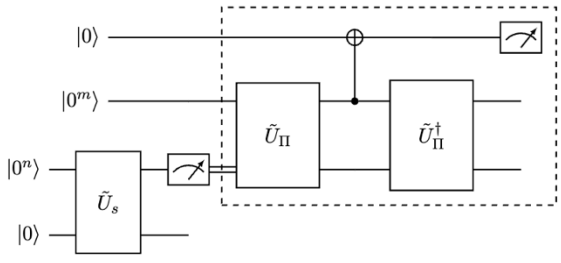

Figure 1. Quantum circuit for estimating the persistent Betti numbers. $\tilde{U}_s$ is a unitary for state preparation and $\tilde{U}_\Pi$ is a unitary encoding of the projector onto the ground space of the persistent Laplacian.

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Stability of Symmetry-Protected Topological Phases as Resources of Measurement-Based Quantum Computation

Physics of Matter: Condensed Matter Physics (YITP) Riku Masui

Abstract We study the stability of symmetry-protected topological phases from the quantum informational viewpoint. The gate fidelity of the measurement-based quantum computation, which quantifies how robust edge states are against quantum noises, provides us with a new insight into the classification of the decohered SPT phases in open quantum systems.

The Haldane phase, which includes the one-dimensional Affleck-Kennedy-Lieb-Tasaki (AKLT) state, serves as a prime example of the symmetry-protected topological (SPT) phases. In one-dimensional quantum systems, SPT phases are distinguished from trivial ones by, e.g., finite string order parameters (SOPs), as well as by the degeneracy of the entanglement spectrum, when the phases are characterized by pure states. Being characterized by short-range entanglement, the SPT phases are applicable to measurement-based quantum computation (MBQC) [1]. This observation led us to explore the use of the MBQC gate fidelity [2] as a tool for detecting SPT phases and studying the stability of them in open quantum systems. This approach enables us to characterize the stability of SPT phases from a quantum computational perspective.

In Ref. [3], two types of symmetries of quantum channels that can either preserve or disrupt the SPT order are discussed, along with the calculation of SOPs. According to their findings, the dephasing noise preserves the SPT order of the Haldane phase, whereas the depolarizing one destroys it. Motivated by these results, we conducted analytical and numerical calculations of the gate fidelity of Z-rotation gates implemented by MBQC on the AKLT state, which is subject to both types of noise-induced decoherence. Our results demonstrate that the behavior of gate fidelity in response to these noises is consistent with the conclusion of Ref. [3]. Moreover, we have shown that the gate fidelity gives us more detailed information on the classification of the decohered SPT phases compared to the previous methods based on the SOP and other order parameters.

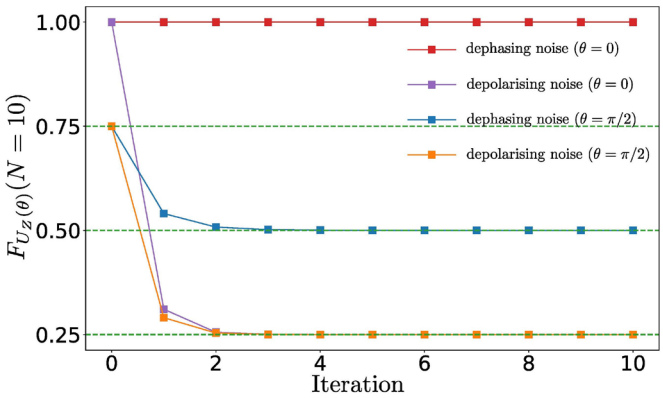

Fig. 1. Numerical results of the fidelity for the gate $U_Z(\theta) = \exp(-i\theta Z/2)$ against two types of noises.

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Dynamic coupling of dynamics of bilayers and Brownian motion of colloidal particles

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Abstract We found “dynamic coupling” between bilayer fluctuations and colloid Brownian motion. For lamellae, we found that the layer compression modulus was reduced because colloidal motions hinder interactions. In contrast, in a sponge phase, the elastic Gaussian curvature modulus of the sponge was reduced. Colloid motions reduce the stability each system.

Lyotropic liquid crystals (LLCs), which are formed by an amphiphilic surfactant and water, adopt a wide variety of structures. For some LLCs, the basic composition unit is a bilayer of surfactant molecules. Typical bilayer phases include the lamellar phase, where bilayers spread two-dimensionally and form a layered structure, and the sponge phase, where bilayers separate the solvent into two regions and create a bicontinuous structure.

Bilayers undulate in water by thermal fluctuations. In lamellae or sponges, some bilayers interact with each other and undergo a cooperative diffusion depending on the structure. In the lamellar phase, adjacent bilayers collide directly: this steric interaction stabilizes lamellae. In the sponges, the positions of regions can change, causing a local change in topology. Undulations of lamellae depend on the layer compression modulus while the topological dynamics of sponges depend on the elastic Gaussian curvature modulus.

Mixtures of bilayers and colloidal particles exhibit a number of unique phenomena due to interactions between its components [1-2]. They focused on static characteristics or response from external field. So in this work, we focus on “dynamical coupling” i.e. between the dynamics of bilayers and the thermal motion of colloidal particles, using dynamic light scattering (DLS) to probe fluctuations. By studying the undulations of lamellae and topological dynamics of sponge, distinct fluctuations originating from the same undulations of bilayer, we study the effect of colloidal motion on the undulation of the bilayer from different points of view.

Firstly, in a mixture of lamellae and colloidal particles, the autocorrelation function from DLS always showed a single relaxation mode corresponding to undulations of lamellae in mixture. We found that the mode slows down monotonically with increasing colloid concentration Φ (Fig. 1). Colloidal motions are coupled with undulations of lamellae, such that adjacent lamellar bilayers collide direct each other by undulations. Then colloidal motions inhibit the direct lamellar collisions and reduce the layer compression modulus.

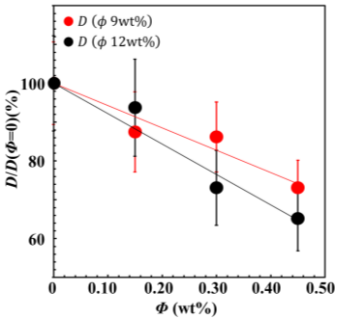

Fig. 1 Diffusion coefficient vs. colloid concentration Φ in a mixture of lamellae and colloidal particles.

Next, in a mixture of sponges and colloidal particles, a single mode corresponding to topological fluctuations of the sponge phase was observed. Unlike for lamellae, the topological dynamics of the sponge becomes *faster* when more colloidal particles are introduced (Fig. 2). This means that colloidal motions help crumple the membrane and reduce the elastic Gaussian curvature modulus.

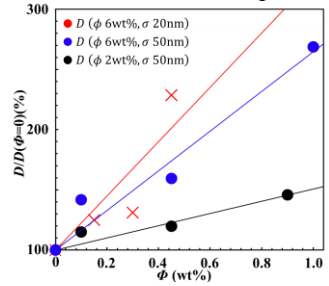

Fig. 2 Diffusion coefficient vs. colloid concentration Φ in a mixture of sponges and colloidal particles.

In each case, colloidal motions reduce the moduli by colliding with undulating bilayers. These moduli stabilize the phase respectively, so colloidal motions reduce the stability of the phase.

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